Quantifying Self-Organization and Coherent Structures with Statistical Complexity

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References

Shalizi, Shalizi & Haslinger, PRL 93 (2004): 118701 = nlin.AO/0409024

Shalizi, DMTCS "AB" (2003): 11 = math.PR/0305160

Shalizi, Rouquier, Haslinger, Moore & Shalizi, in prep.

- “I know it when I see it”
- Disputes: turbulence, ecology
- Does self-organizing ⇒ irreversible?
  Yes: Priogine, Nicolis; Haken; etc.
  No: D’Souza, Margolus; Smith
- *Not* self-organized criticality (necessarily)
- Why not just use entropy?

Entropy

- Entropy = log of volume in phase space

$$S = k \log W$$

- Much to do with heat, work, etc., etc.
- Nothing to do with organization
    - Low entropy disorganized systems (low-temperature stat. mech.)
    - High entropy organized things (organisms)
    - Organization ↑ *because* entropy ↑ (self-assembly)

If Not Entropy, What?

- Self-organization = complexity increases
- Complexity = description length (Kolmogorov, von Neumann, etc.)
- Probabilistic descriptions, statistical complexity (Rissanen, many others)
- Complexity = information required for prediction (Grassberger)
- Role of minimal sufficient statistics (Crutchfield & Young)

Local Statistics

- Local statistics are summaries of the past light-cone
- Maximize information in statistic (sufficiency) ⇒ maximally accurate prediction
- Minimize information needed by statistic (minimality)

- past A and past B equivalent iff

$$\Pr(\text{Future}|A) = \Pr(\text{Future}|B)$$

- [A] = all pasts equivalent to A
- Statistic ("causal state"):

$$\varepsilon(\text{past}) = [\text{past}]$$

- Sufficient, minimal, unique, recursive, etc.
- Complexity = information needed to fix state

$$C = I[\text{past};\varepsilon(\text{past})]$$

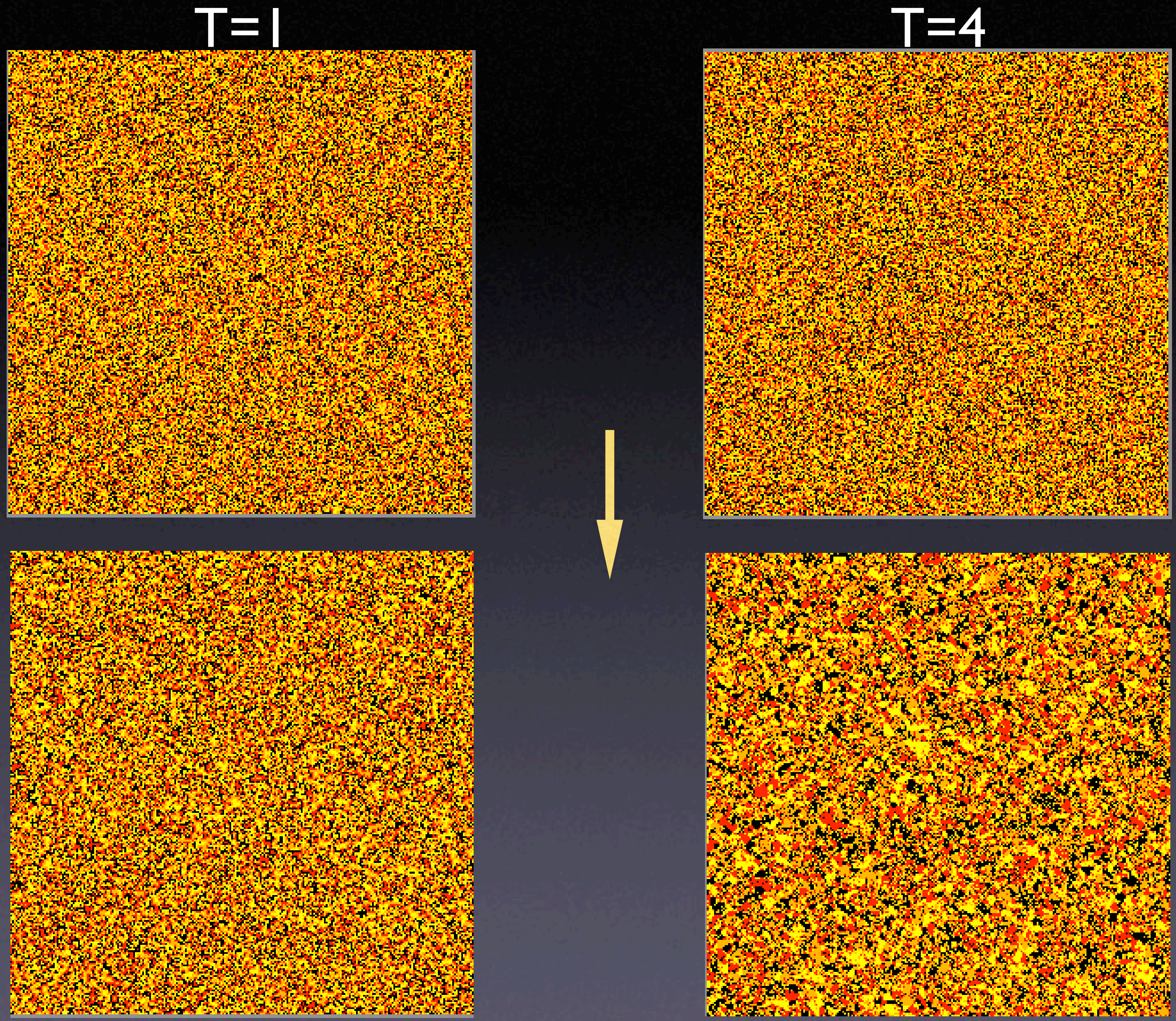
System has self-organized between t_1 and t_2 if

(I) $C(t_1) < C(t_2)$

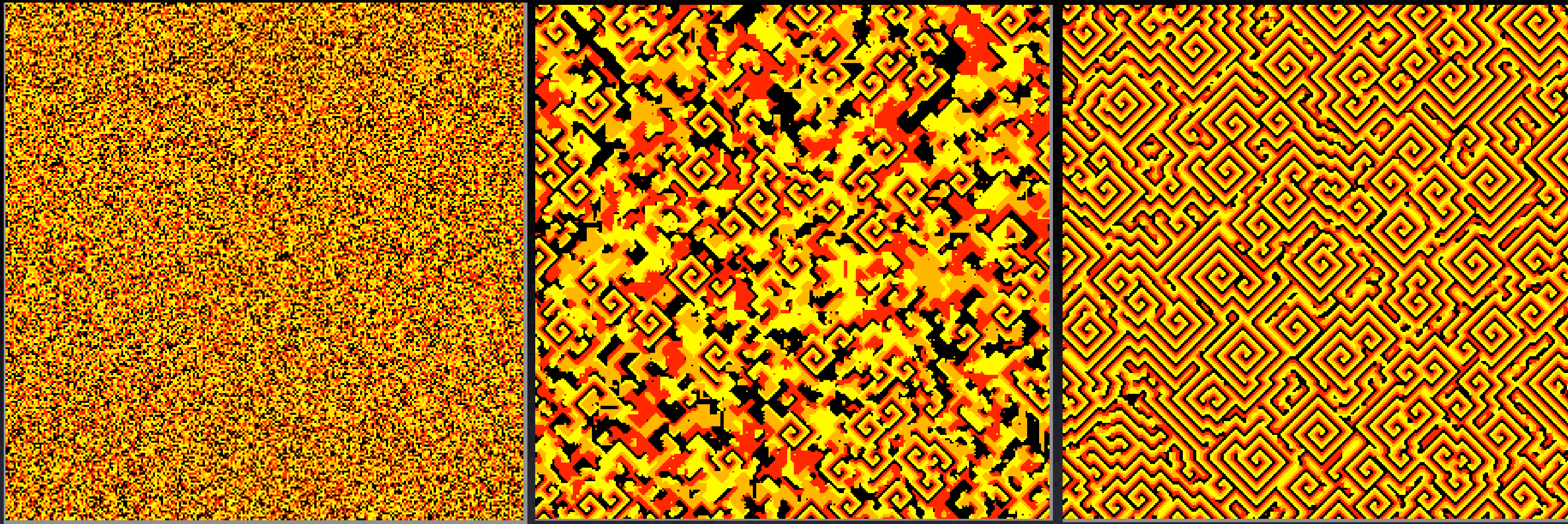
(II) the increase is not caused by outside input

Cyclic Cellular Automata

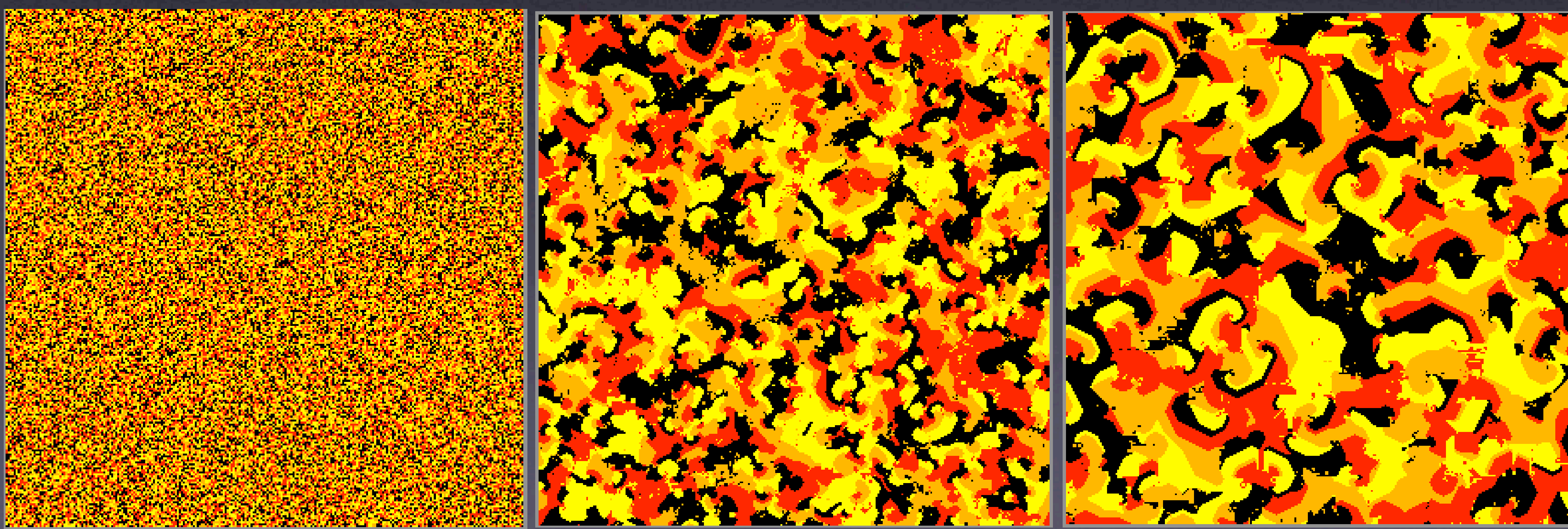
- Qualitative model of excitable media
- K colors; a cell of color k switches to k+1 (mod K) if at least T neighbors are already of that color
- Analytic theory for structures formed (Griffeath et al.)
- Spirals, "turbulence", local oscillation, fixation


T=1
T=4

T=2



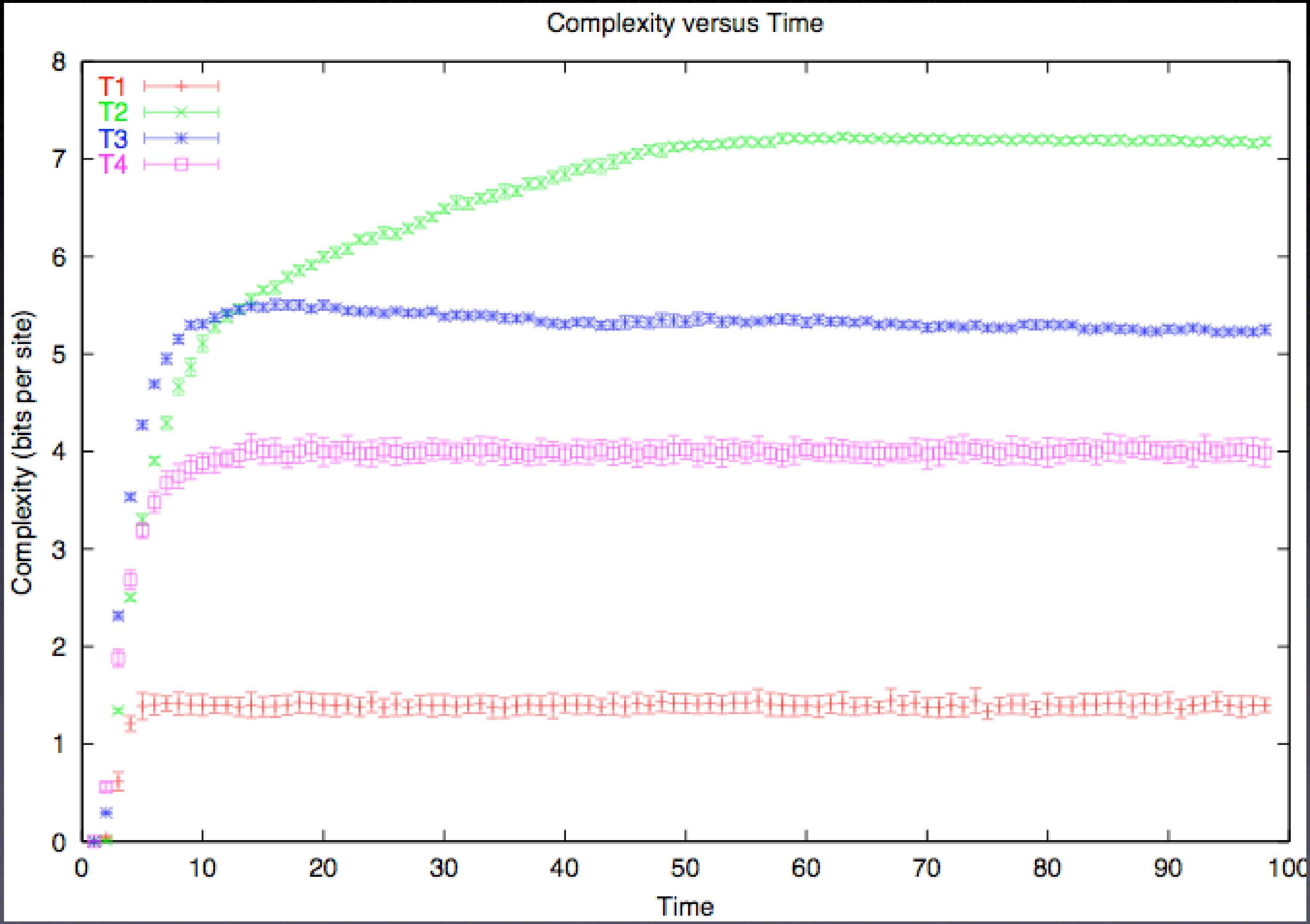
T=3



Identification of states from data

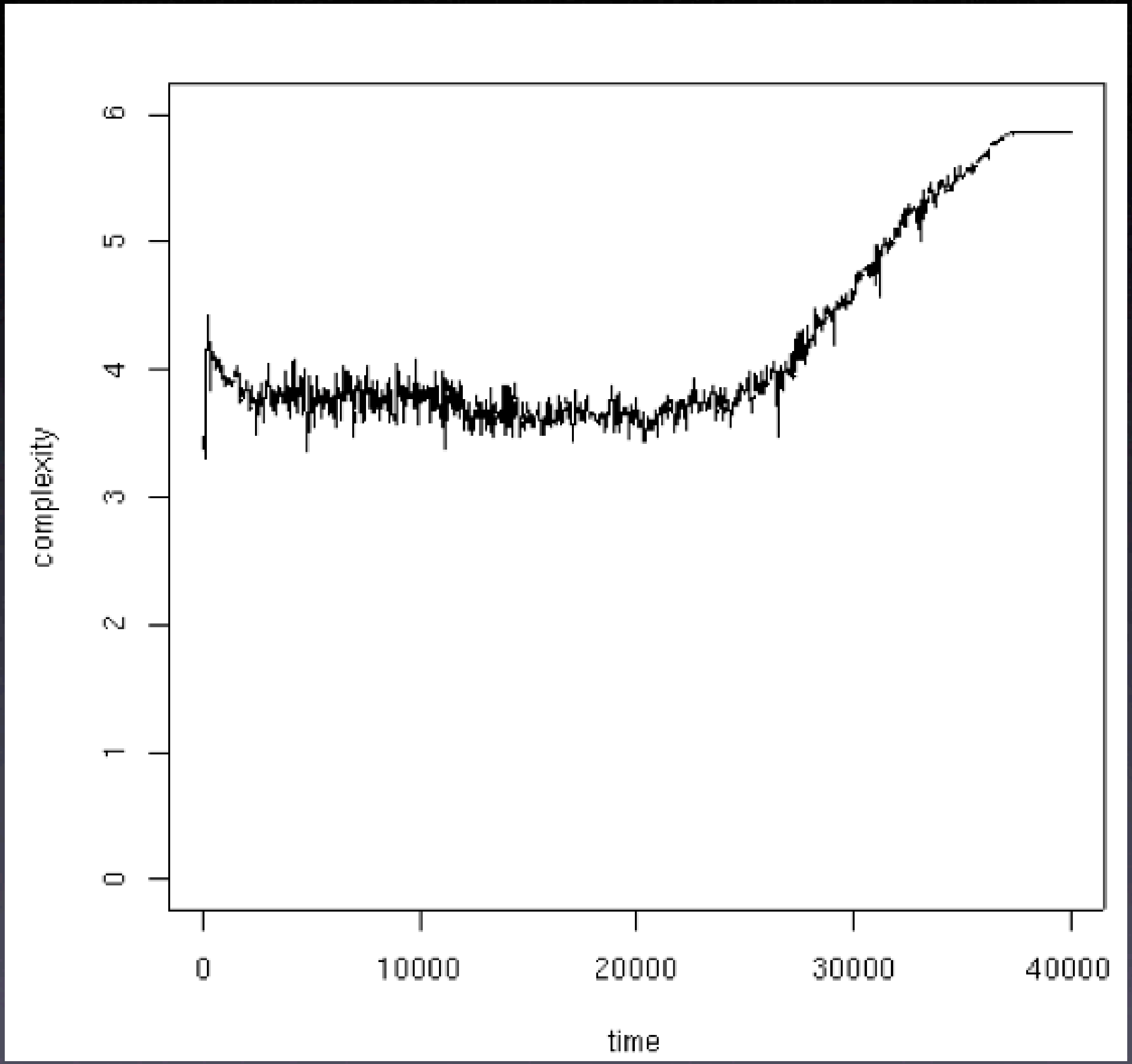
- Clustering of past light-cone configurations by similarity of empirical distributions for future
- Consistent estimates of states, C(t)

CCA



(300x300, n = 30)

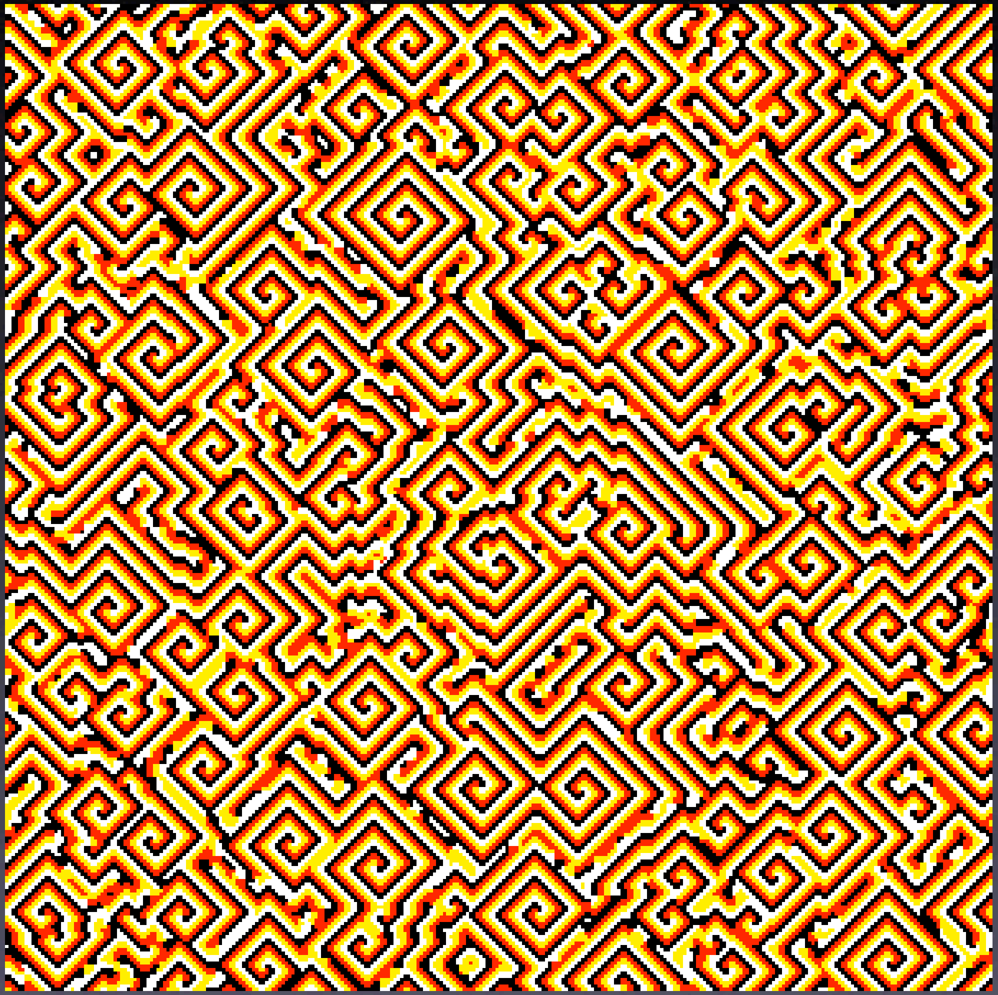
BTW Sandpile



(supra-threshold relaxation, 300x300, n=1)

Finding Coherent Structures

- Spatially extended, temporally persistent
- Generated by the micro dynamics ("emergent")
- More efficient and more comprehensible descriptions



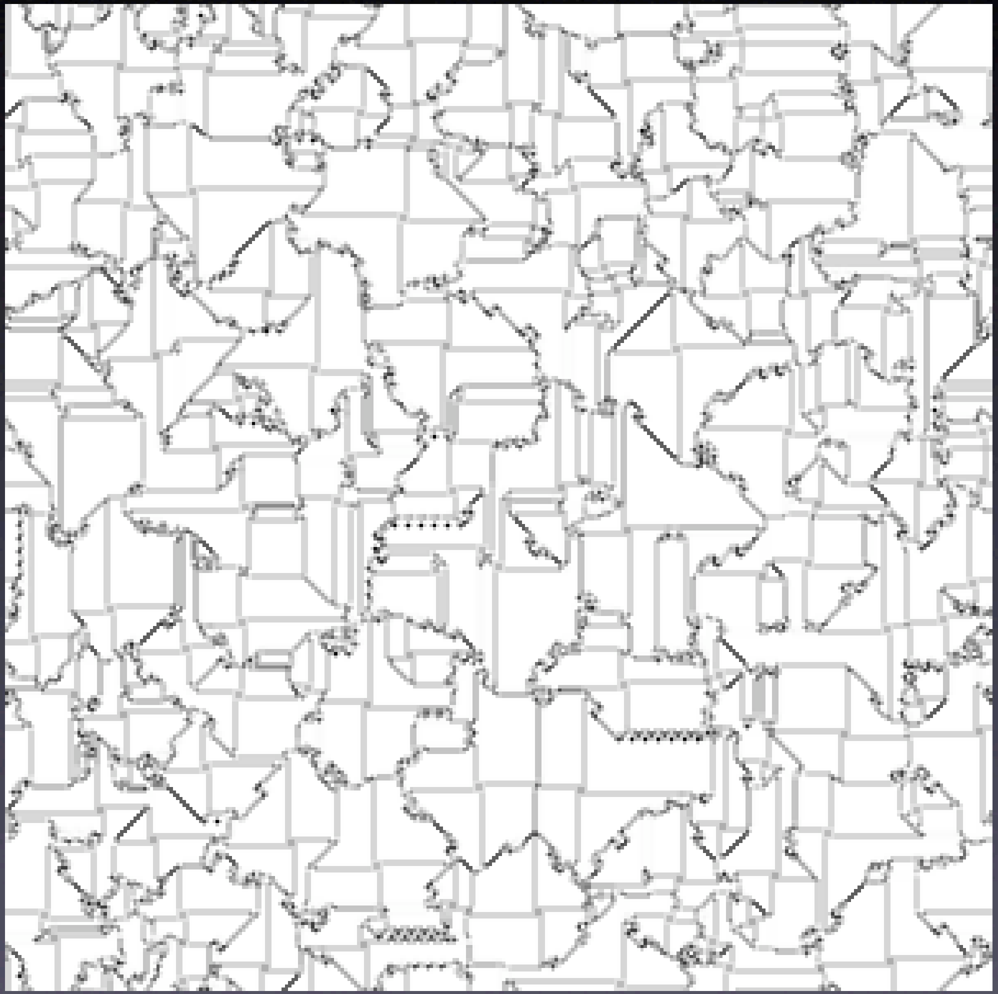
Order parameters

- OP measures symmetry breaking

$$\Phi = f(OP)$$

$$-\log(\Pr(\text{config})) \sim \int f(OP)$$

- Structures = defects in OP field
- OP found by trial and error

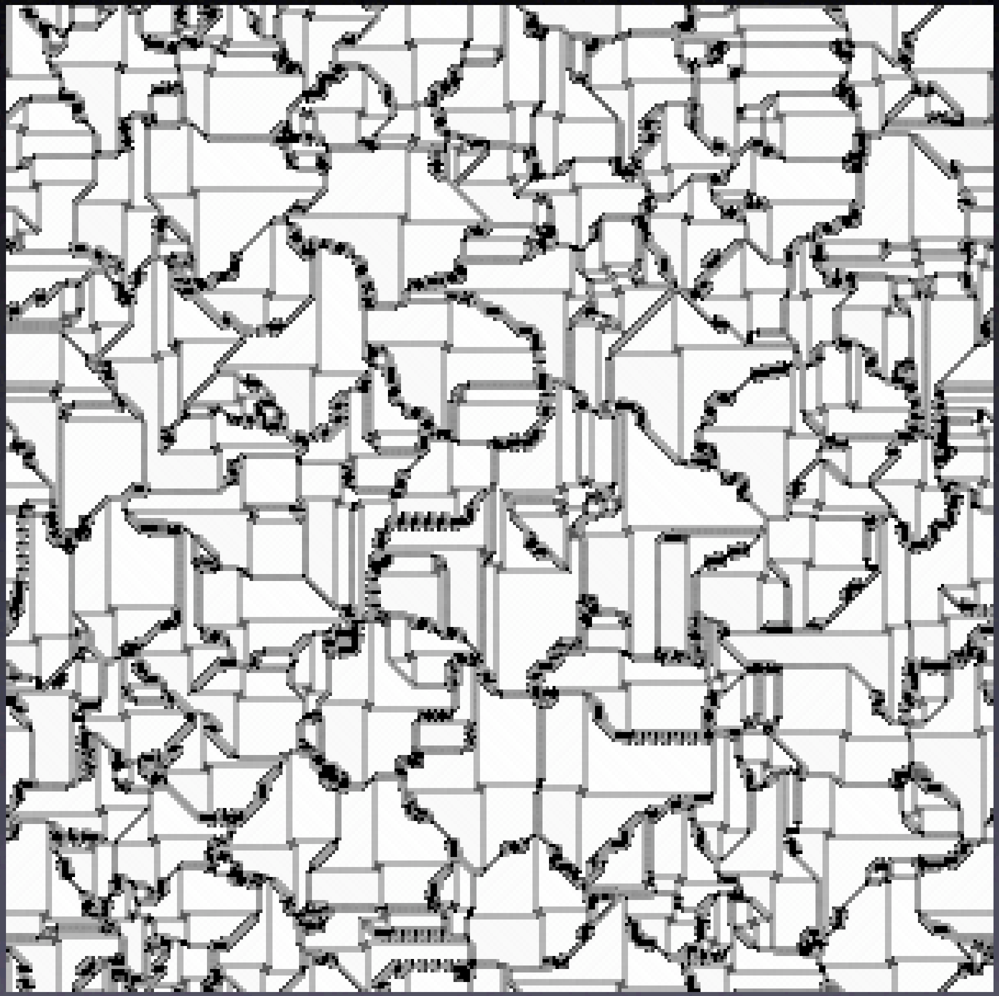


Complexity field

- Local description length

  $C = -\log(\Pr(\text{state}))$

- Automatic; no tradition needed



Summary

- Statistical complexity = information needed for optimal prediction
- Self-organization = internally-caused rise in complexity
- This definition works in practice
- The local complexity field lets us identify coherent structures automatically